CALCULATION OF TEMPERATURE FIELDS IN THERMAL DRILLING OF GLACIAL DEPOSITS

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The method of moving heat sources [1, 2] is applied to the calculation of temperature fields in drilling glacial deposits with a hot disc-shaped drill.

Let us consider the adiabatic surface of a semi-infinite body (Fig. 1), on which a plane circular heat source of strength q and radius R_0 begins to act at time $\tau = \tau_1 = 0$ moving at a constant velocity v into the depths of the body, which has thermophysical coefficients λ, $c\gamma$ and a, and the initial temperature T_0, taken as an arbitrary zero reading. The moving source continues to act for a time $\tau = \tau_0$. Subsequent motion of the source takes place without the release of heat. It is required to determine the temperature field in the moving coordinate system x0yz. If it is assumed that in time τ_0 the source moves a certain distance away from the top boundary of the semi-infinite body, then the influence of this boundary on the distribution of heat may be neglected, and the source can be assumed to move in an infinite body.

We shall find the solution of the problem, using the equation of the temperature field for an instantaneous moving point source of heat $0'(x_0, 0, z_0)$ of strength q_1, which, for a moving system of polar coordinates, may be written in the following form [3]:

$$T = \frac{q}{8c\gamma(\pi a)^{3/2}(\tau-\tau_1)^{3/2}} \times$$

$$\times \exp\left[-\frac{R_{xz}^2 + R_1^2 - 2R_{xz}R_1\cos\varphi + [y + v(\tau-\tau_1)]^2}{4a(\tau-\tau_1)}\right]. \tag{1}$$

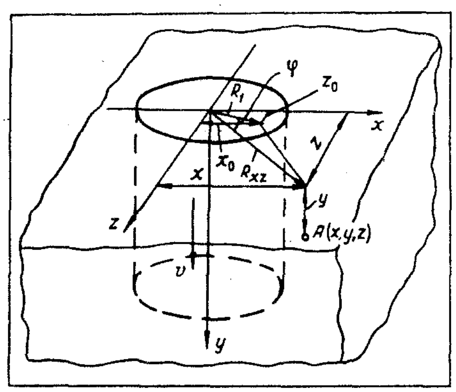

Fig. 1. Diagram illustrating motion of a plane circular heat source in a semi-infinite body.

In obtaining the solution for the temperature field due to the moving circular heat source q, the right hand side of (1) must be multiplied by R_1, and the resulting expression integrated with respect to φ within the limits 0 to 2π, and also with respect to R_1 and τ_1 within the limits 0 to R_0 and $\tau_1 = 0$ to $\tau_1 = \tau_0$, respectively.

In the general case we have

$$T = \frac{q}{8c\gamma(\pi a)^{3/2}} \int\limits_{\tau_1=0}^{\tau_1=\tau_0} \frac{d\tau_1}{(\tau-\tau_1)^{3/2}} \int\limits_0^{R_0} R_1 dR_1 \int\limits_0^{2\pi} \times$$

$$\times \exp\left[-\frac{R_{xz}^2 + R_1^2 - 2R_{xz}R_1\cos\varphi + [y+v(\tau-\tau_1)]^2}{4a(\tau-\tau_1)}\right] d\varphi. \tag{2}$$

Consider only the temperature field along the y axis ($R_{xz} = 0$), we obtain, after integration with respect to φ and R_1,

$$T = \frac{q}{2\sqrt{\pi\lambda c\gamma}} \exp\left(-\frac{vy}{2a}\right)\left[\int\limits_{\tau-\tau_0}^{\tau} \exp\left[-\left(\frac{m}{u} + nu\right)\right]\frac{du}{\sqrt{u}} - \right.$$

$$\left. - \int\limits_{\tau-\tau_0}^{\tau} \exp\left[-\left(\frac{k}{u} + nu\right)\right]\frac{du}{\sqrt{u}}\right], \tag{3}$$

where

$$m = \frac{y^2}{4a},\ n = \frac{v^2}{4a},\ k = \frac{R^2}{4a} \quad \text{and} \quad u = \tau - \tau_1.$$

By substituting

$$x_1 = \sqrt{\frac{m}{u}} + \sqrt{nu},\ x_2 = \sqrt{\frac{m}{u}} - \sqrt{nu},$$

$$z_1 = \sqrt{\frac{k}{u}} + \sqrt{nu} \ \text{и}\ z_2 = \sqrt{\frac{k}{u}} - \sqrt{nu}$$

followed by integration of (3) by parts, we obtain the final equation of the temperature field for the period of cooling ($\tau > \tau_0$) in the form

$$T = \frac{q}{2c\gamma v}\left\{\left[\operatorname{erf}\left(\frac{y}{2\sqrt{a\tau}} + \frac{v\sqrt{\tau}}{2\sqrt{a}}\right) - \right.\right.$$

$$- \operatorname{erf}\left(\frac{y}{2\sqrt{a(\tau-\tau_0)}} + \frac{v\sqrt{\tau-\tau_0}}{2\sqrt{a}}\right)\Big] - \exp\left(-\frac{vy}{a}\right) \times$$

$$\times\left[\operatorname{erf}\left(\frac{y}{2\sqrt{a\tau}} - \frac{v\sqrt{\tau}}{2\sqrt{a}}\right) - \operatorname{erf}\left(\frac{y}{2\sqrt{a(\tau-\tau_0)}} - \right.\right.$$

$$\left.\left. - \frac{v\sqrt{\tau-\tau_0}}{2\sqrt{a}}\right)\right] - \exp\left(\frac{v(R-y)}{2a}\right) \times$$

$$\times\left[\operatorname{erf}\left(\frac{R}{2\sqrt{a\tau}} + \frac{v\sqrt{\tau}}{2\sqrt{a}}\right) - \operatorname{erf}\left(\frac{R}{2\sqrt{a(\tau-\tau_0)}} + \right.\right.$$

$$\left.\left. + \frac{v\sqrt{\tau-\tau_0}}{2\sqrt{a}}\right)\right] + \exp\left(-\frac{v(R+y)}{2a}\right) \times$$

$$\times\left[\operatorname{erf}\left(\frac{R}{2\sqrt{a\tau}} - \frac{v\sqrt{\tau}}{2\sqrt{a}}\right) - \right.$$

$$\left.\left.- \operatorname{erf}\left(\frac{R}{2\sqrt{a(\tau-\tau_0)}} - \frac{v\sqrt{\tau-\tau_0}}{2\sqrt{a}}\right)\right]\right\}. \qquad (4)$$

The temperature field at the end of the heating phase ($\tau = \tau_0$) is given by the equation

$$T = \frac{q}{2c\gamma v}\left\{\exp\left(-\frac{vy}{a}\right)\operatorname{erfc}\left(\frac{y}{2\sqrt{a\tau_0}} - \right.\right.$$

$$\left. - \frac{v\sqrt{\tau_0}}{2\sqrt{a}}\right) - \operatorname{erfc}\left(\frac{y}{2\sqrt{a\tau_0}} + \frac{v\sqrt{\tau_0}}{2\sqrt{a}}\right) +$$

$$+ \exp\left[\frac{v(R-y)}{2a}\right]\operatorname{erfc}\left(\frac{R}{2\sqrt{a\tau_0}} + \frac{v\sqrt{\tau_0}}{2\sqrt{a}}\right) -$$

$$\left. - \exp\left[-\frac{v(R+y)}{2a}\right]\operatorname{erfc}\left(\frac{R}{2\sqrt{a\tau_0}} - \frac{v\sqrt{\tau_0}}{2\sqrt{a}}\right)\right\}. \qquad (5)$$

The quasi-steady temperature field ($\tau_0 = \infty$) has the form

$$T = \frac{q}{c\gamma v}\exp\left(-\frac{vy}{a}\right)\left[1 - \exp\left(-\frac{v(R-y)}{2a}\right)\right] \quad \text{for} \quad y \geqslant 0, \qquad (6)$$

$$T = \frac{q}{c\gamma v}\left[1 - \exp\left[-\frac{v(R-|y|)}{2a}\right]\right] \quad \text{for} \quad y \leqslant 0. \qquad (7)$$

For the temperature at the center of the source (y = 0), the following expressions are obtained:

1. Cooling period ($\tau > \tau_0$),

$$T = \frac{q}{2c\gamma v}\left\{2\left[\operatorname{erf}\left(\frac{v\sqrt{\tau}}{2\sqrt{a}}\right) - \operatorname{erf}\left(\frac{v\sqrt{\tau-\tau_0}}{2\sqrt{a}}\right)\right] - \exp\left(\frac{vR_0}{2a}\right) \times\right.$$

$$\times\left[\operatorname{erf}\left(\frac{R_0}{2\sqrt{a\tau}} + \frac{v\sqrt{\tau}}{2\sqrt{a}}\right) - \operatorname{erf}\left(\frac{R_0}{2\sqrt{a(\tau-\tau_0)}} + \right.\right.$$

$$\left.\left. + \frac{v\sqrt{\tau-\tau_0}}{2\sqrt{a}}\right)\right] + \exp\left(-\frac{vR_0}{2a}\right) \times$$

$$\times\left[\mathrm{erf}\left(\frac{R_0}{2\sqrt{a\tau}}-\frac{v\sqrt{\tau}}{2\sqrt{a}}\right)-\mathrm{erf}\left(\frac{R_0}{2\sqrt{a(\tau-\tau_0)}}-\frac{v\sqrt{\tau-\tau_0}}{2\sqrt{a}}\right)\right]\Big\}. \qquad (8)$$

2. End of heating phase ($\tau = \tau_0$),

$$T=\frac{q}{2c\gamma v}\left[2\,\mathrm{erf}\left(\frac{v\sqrt{\tau_0}}{2\sqrt{a}}\right)+\exp\left(\frac{vR_0}{2a}\right)\times\right.$$

$$\times\,\mathrm{erfc}\left(\frac{R_0}{2\sqrt{a\tau_0}}+\frac{v\sqrt{\tau_0}}{2\sqrt{a}}\right)-$$

$$\left.-\exp\left(-\frac{vR_0}{2a}\right)\mathrm{erfc}\left(\frac{R_0}{2\sqrt{a\tau_0}}-\frac{v\sqrt{\tau_0}}{2\sqrt{a}}\right)\right]. \qquad (9)$$

3. Quasi-steady phase ($\tau_0 = \infty$),

$$T=\frac{q}{c\gamma v}\left[1-\exp\left(-\frac{vR_0}{2a}\right)\right]. \qquad (10)$$

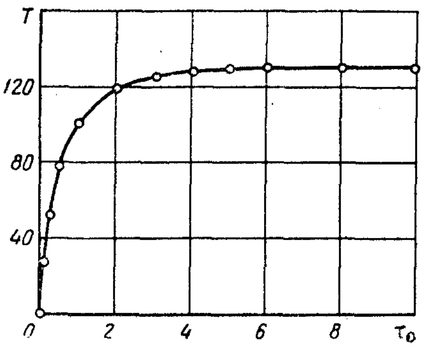

Fig. 2. Variation with time of temperature at center of plane circular heat source ($v = 0.228$ m/sec, $\lambda = 55.1$ w/m · degree, $a = 1.31 \cdot 10^{-3}$ m²/sec, $\Delta T = 130$°C, $Q = 10$ kw, $R_0 = 0.05$ m)

The heat emitted by the source in unit time is

$$Q = qF = \pi q R_0^2.$$

Therefore

$$q = Q/\pi R_0^2. \qquad (11)$$

The results of calculation of temperature T at the center of the source ($y = 0$) from (9) and (11) are given in Fig. 2, and show that in hot drilling of glacial strata the quasi-steady phase begins in practice not when $\tau_0 \to \infty$, but 3 to 4 sec after the heat source starts to move. We may therefore use (6), (7), and (10) for practical calculations with a sufficient degree of accuracy. Taking into account (11), these expressions take the form

$$\frac{2\pi\lambda R_0 T}{Q}=\frac{2a}{vR_0}\exp\left(-\frac{vy}{a}\right)\times$$

$$\times\left[1-\exp\left(-\frac{v(R-y)}{2a}\right)\right] \text{ for } y>0, \qquad (12)$$

$$\frac{2\pi\lambda R_0 T}{Q}=\frac{2a}{vR_0}\left[1-\exp\left(-\frac{v(R-|y|)}{2a}\right)\right] \text{ for } y<0, \qquad (13)$$

$$\frac{2\pi\lambda R_0 T}{Q}=\frac{2a}{vR_0}\left[1-\exp\left(-\frac{vR_0}{2a}\right)\right] \text{ for } y=0. \qquad (14)$$

The main equation for calculating the process of hot drilling of glacial strata is (14), which makes it possible to calculate the power of the source Q for a given drilling speed v, and conversely the drilling speed for a given source power.

The graphical relationship between the dimensionless groups $2\pi\lambda R_0 T/Q$ and $vR_0/2a$ is shown in Fig. 3. It indicates that, as $vR_0/2a$ changes from 0 to ∞, the value of the quantity $2\pi\lambda R_0 T/Q$ decreases from 1 to 0, respectively. For constant R_0 and Q, an increase in v always causes a decrease in source temperature T, and vice versa.

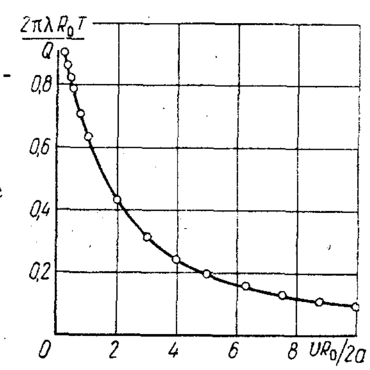

Fig. 3. Graph of the relation

$$\frac{2\pi R_0 T}{Q} = f\left(\frac{vR_0}{2a}\right)$$

NOTATION

τ – time; τ_0 – time of action of heat source; q – strength of plane circular source; q_1 – strength of point heat source; R_0 – radius of plane circular source; $R_1 = \sqrt{x_0^2+y_0^2}$ – position radius of point heat source; v – velocity of source (drilling speed); λ – thermal conductivity; $c\gamma$ – heat capacity per unit volume; α – thermal diffusivity; T_0 – initial temperature of material; $\Delta T = T - T_0$ – instantaneous temperature difference; erfc $(x) = 1 -$ erf (x), where erf $(x) = \frac{2}{\sqrt{\pi}} \cdot \int_0^x \exp(-x^2)dx$ is the Gaussian error function; x, y, z is the moving system of coordinates, $R = \sqrt{R_0^2 + y^2}$.

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